



# **Efficient Testing Combining Design of Experiment and Learn-to-Fly Strategies**

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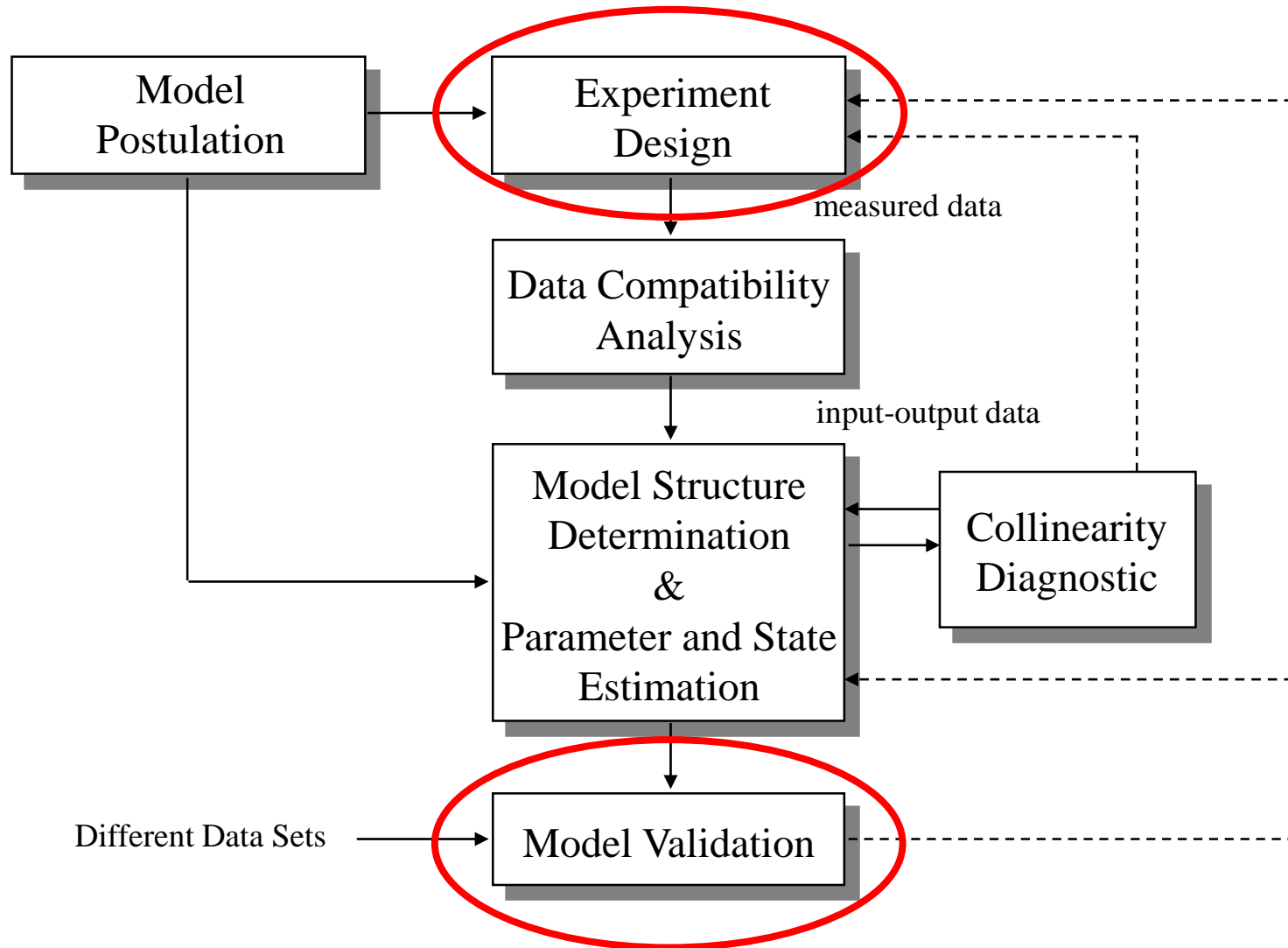
# Outline

- Introduction
  - Seeking greater efficiency & performance through experiment design
    - Efficiency gained by collecting the “right amount” of data
    - Performance gained by adding statistical rigor
- System Identification Process in Wind Tunnel
  - Design of Experiment (DOE)
  - Learn-to-Fly (L2F)
  - Blended DOE-L2F
    - First time testing blended concept – strawman approach
    - Work in progress
- Analysis, Results, and Validation Tests
  - DOE Tests
  - L2F Tests
  - Blended DOE-L2F Tests
- Concluding Remarks



- Wide spectrum of modeling demands
  - Fidelity requirements
  - Aircraft complexity
- Aircraft complexity drive up costs
  - Conventional practice in LaRC 12-foot Wind Tunnel (static test)
    - 100 Hz sample rate, dwell for 10 seconds, average data
    - ~ 2 data pts/min
  - Simple factorial test for L-59
    - 9-Factors:  $\alpha$ ,  $\beta$ , and 7 control surfaces
    - $2^9 = 512$  test points  $\Rightarrow$  4.26 hours
  - Reasonable data density often requires  $5^9 \Rightarrow$  16,276 hours (~8years)!
- Investigators must tradeoff of cost vs fidelity/complexity
  - Define purpose of model and required fidelity. What is allowable error?
  - Asking for “best possible answer” is not adequate
  - Speeding up the modeling process helps anywhere on spectrum

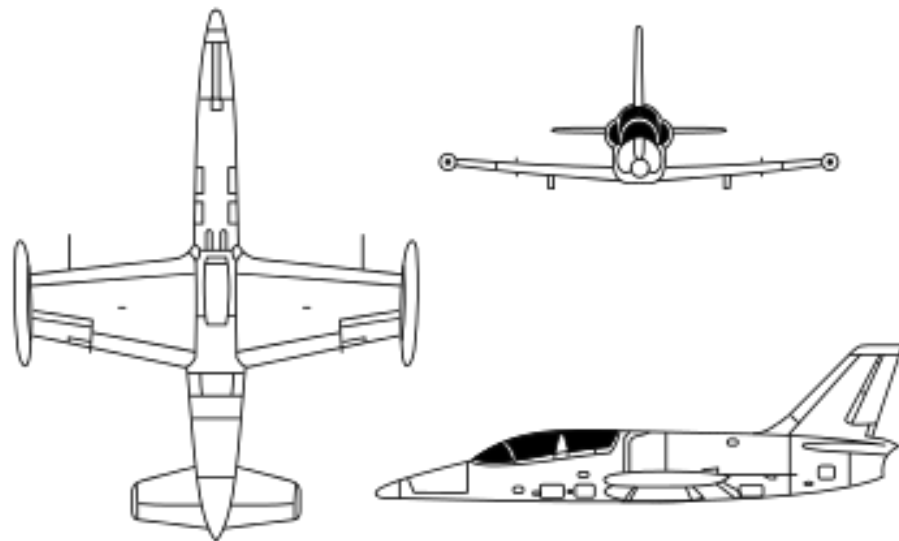
# Aircraft System Identification Process



# Test vehicle for Wind Tunnel Static Test

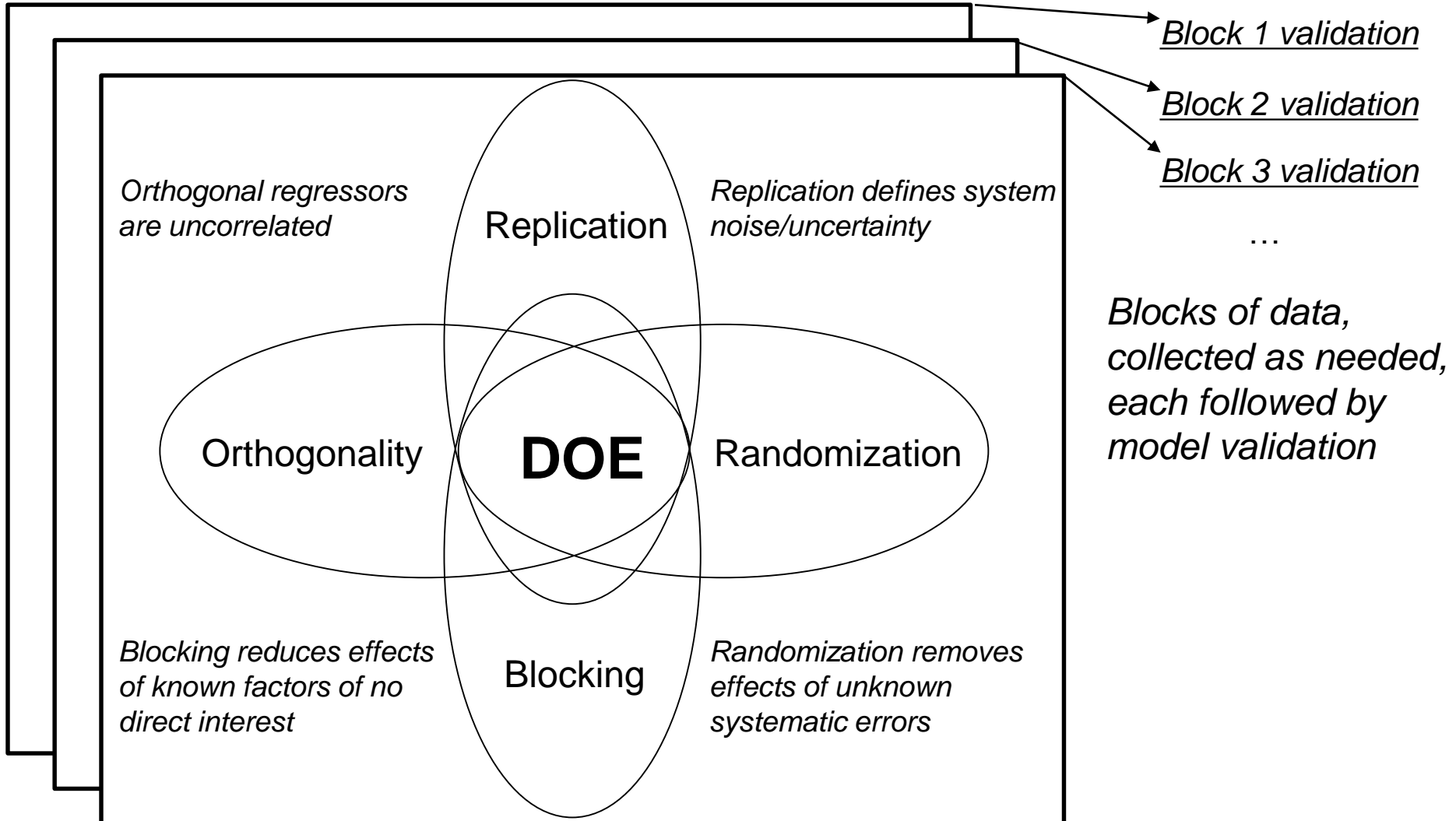
No.	Label	Description	Low Value	High Value	Units
1	aoa	Aircraft alpha	-2	20	deg
2	beta	Aircraft beta	-5	5	deg
3	dela_L	Aileron left wing	-25	25	deg
4	dela_R	Aileron right wing	-25	25	deg
5	delf_L	Flap left wing	0	40	deg
6	delf_R	Flap right wing	0	40	deg
7	delr	Rudder	-30	30	deg
8	dele_L	Elevator left wing	-30	30	deg
9	dele_R	Elevator right wing	-30	30	deg

- L-59 Albatros
- Czech military trainer
- Low-cost off-the-shelf kit
- 12.5% scale model
- Sport application, RC actuators



# Tenets of Design of Experiment (DOE)

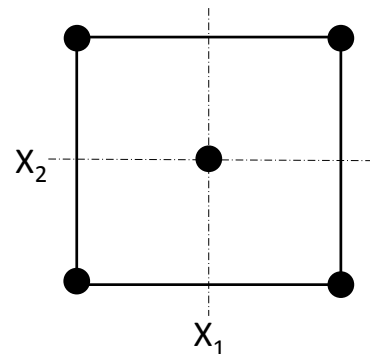
*Sequential testing proceeds only as model complexity requires*



# Block Designs & Supported Models

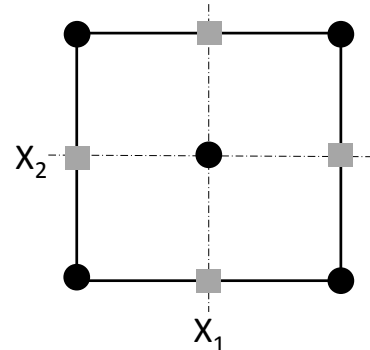
- Full factorial design

$$y = B_0 + \sum_i B_i x_i + \sum_{i \neq j} \sum B_{ij} x_i x_j + \varepsilon \quad i = 1, 2, \dots, k$$



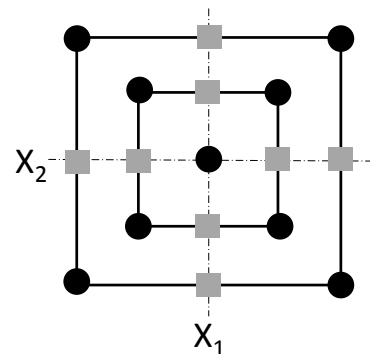
- Face-centered design (FCD)

$$y = B_0 + \sum_i B_i x_i + \sum_i B_{ii} x_i^2 + \sum_{i \neq j} \sum B_{ij} x_i x_j + \varepsilon \quad i = 1, 2, \dots, k$$



- Nested face-centered design

$$y = B_0 + \sum_i B_i x_i + \sum_i B_{ii} x_i^2 + \sum_{i \neq j} \sum B_{ij} x_i x_j + \sum_i B_{iii} x_i^3 + \varepsilon \quad i = 1, 2, \dots, k$$



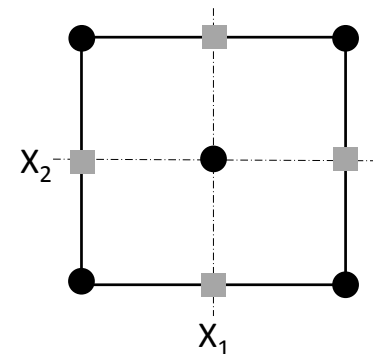
# Block 1, DOE Design Metrics (9-factors)

Block Type	Blocks	Runs	Design Terms	VIF	% Power
	(inclusive)				$2\sigma, s/n=2$
¼ Fraction FCD	1	156	Quadratic	9.64	84.4

Maximum Variance Inflation Factor (VIF),  
reflects lack of orthogonality in design; desire  $\leq 10$

% Power reflects statistical power of design,  
manages type-2 error; desire  $\geq 80$

$$y = B_0 + \sum_i B_i x_i + \sum_i B_{ii} x_i^2 + \sum_{i \neq j} B_{ij} x_i x_j + \varepsilon \quad i = 1, 2, \dots, k$$

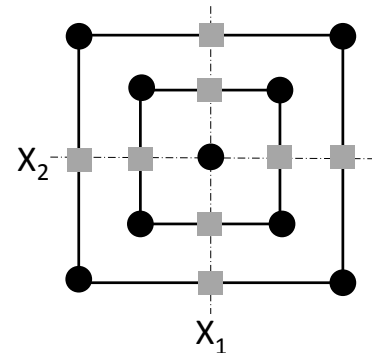


**Validation Test Performed after each block of data**

# Block 2 added to create Nested FCD

Block Type	Blocks (inclusive)	Runs	Design Terms	VIF	% Power $2\sigma, s/n=2$
1/4 Fraction FCD	1	156	Quadratic	9.64	84.4
Nested FCD	1, 2	312	Quadratic	22.41	86.8

$$y = B_0 + \sum_i B_i x_i + \sum_i B_{ii} x_i^2 + \sum_{i \neq j} \sum B_{ij} x_i x_j + \sum_i B_{iii} x_i^3 + \varepsilon \quad i = 1, 2, \dots, k$$



**Require optimized design points to reduce VIF**



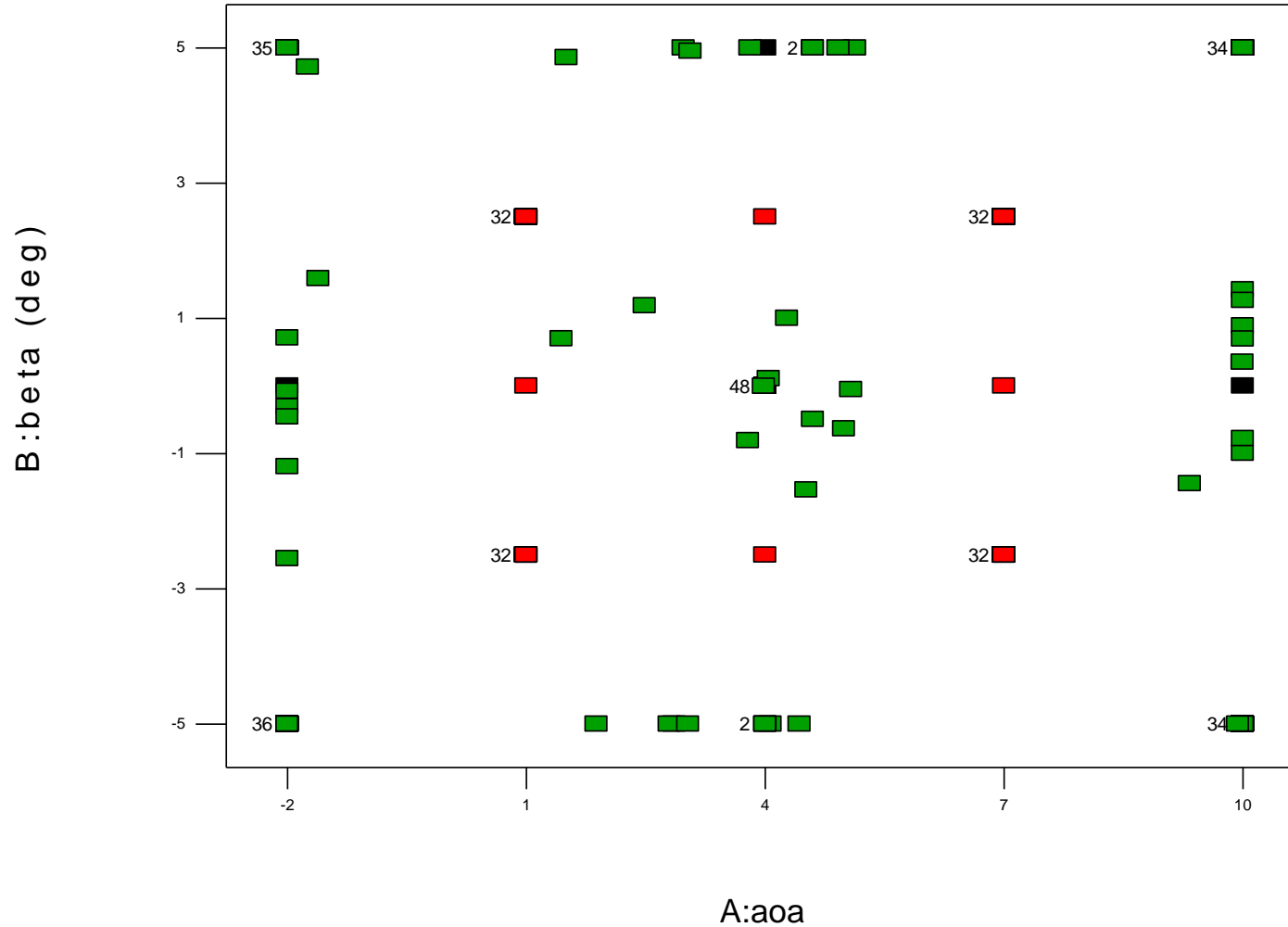
# ***Final DOE Design, 3-blocks***

Block Type	Blocks	Runs	Design Terms	VIF	% Power
	(inclusive)				$2\sigma$ , $s/n=2$
¼ Fraction FCD	1	156	Quadratic	9.64	84.4
Nested FCD	1, 2	312	Quadratic	22.41	86.8
I-optimal	1, 2, 3	366	Quadratic	4.0	99.9

I-optimal block provides test points that minimize prediction error

***Validation Test Performed after each block of data***

# DOE Design for 3 blocks



- FCD (black)
- Nested FCD (red)
- I-optimal (green)

# Stepwise Regression Modeling

- Stepwise regression used to select model parameters

$$y = \beta_0 + \sum_i \beta_i x_i + \sum_i \beta_{ii} x_i^2 + \sum_{i \neq j} \sum \beta_{ij} x_i x_j + \sum_i \beta_{iii} x_i^3 + \dots + \varepsilon \quad i = 1, 2, \dots, 23$$

- Primary metrics utilized for model selection:
  - Stepwise Regression significance level: 95% – 99%
  - Standard ANOVA table analysis
  - Lack of Fit (LOF) measure of model error relative to pure error
  - Standard deviation (fit error)
  - PRESS (prediction error sum of squares)
  - Coefficient of Variation (C.V.% = std. dev. / mean)
  - $e_i / C_{N\max} \%$ ; ( $e_i = C_{N\text{measured}} - C_{N\text{predicted}}$ ) ... desire  $\leq 3\%$
  - $R^2$  , adjusted  $R^2$  , predicted  $R^2$  , (family of metrics)

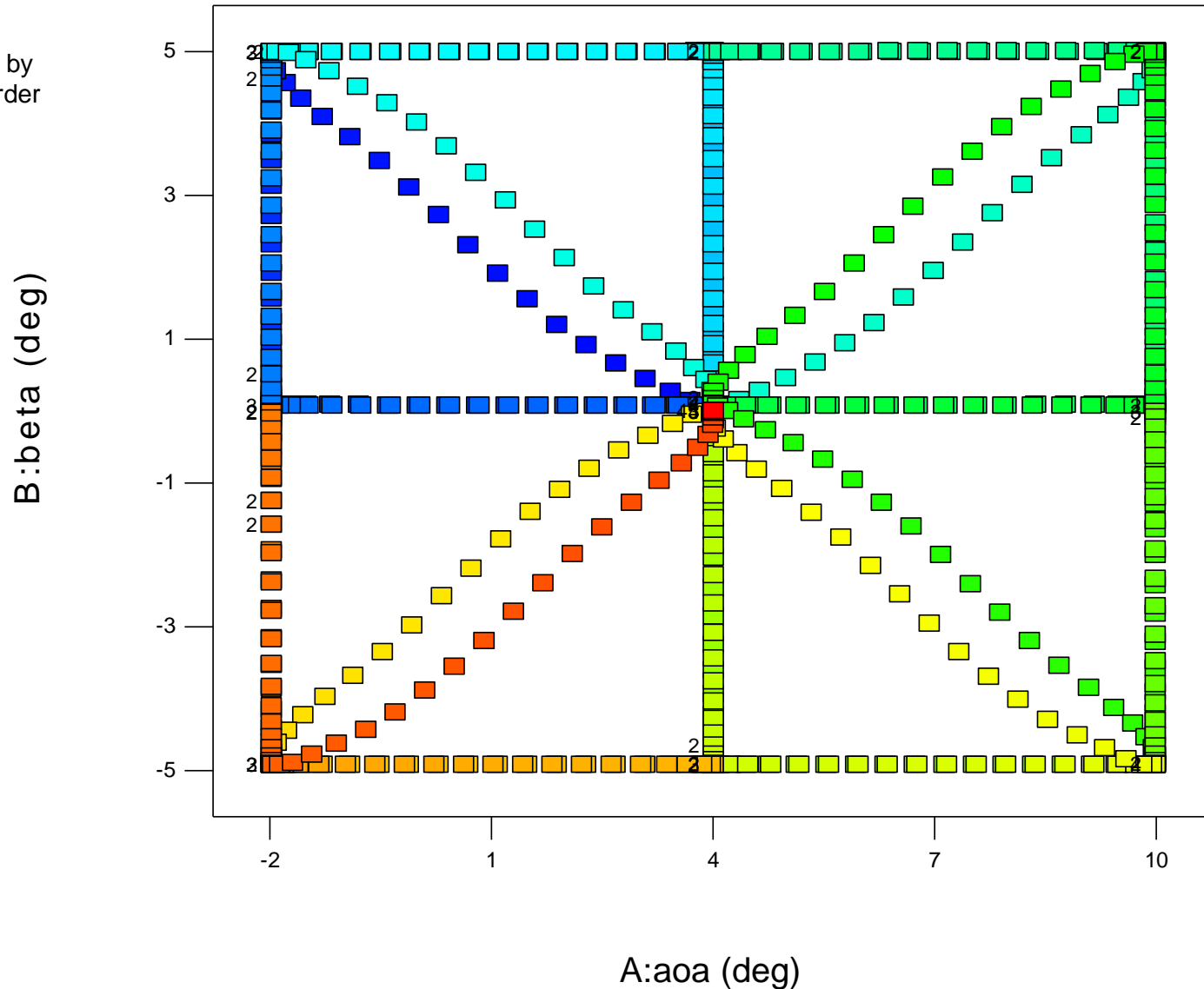
$$R^2 = \frac{\text{variation explained}}{\text{total variation}}; \quad 0 < R^2 < 1$$



# ***Learn-to-Fly (L2F) Testing***

- L2F approach adapted to wind tunnel
  - General L2F approach is real-time global modeling of aerodynamics
  - Applicable to wind tunnel or flight testing
  - Continuous sampling during dynamic test
- This study is a “quasi-static” test
  - Continuous sampling while sweeping target points slowly
  - Batch processing, stepwise regression
- Key to efficiency: Wide-band orthogonal inputs
  - Higher bandwidth (HBW) inputs applied to control surfaces
  - Lower bandwidth (LBW) inputs apply to other factors
- L2F experiment design
  - Test grid is setup for LBW factors
  - LBW trajectories form a nested “FCD-like” design

# Learn-to-Fly (L2F) Trajectories





# Blended DOE-L2F Testing (“quasi-static” test)

- Use key “efficiency features” of both approaches
  - DOE: 4 tenets, sequential testing blocks of data, with validation tests
  - L2F: HBW design for factors that accept wide-band inputs
- Blended design both simplifies and complicates final design
  - Simplifies 9-LBW experiment to a 2-LBW + 7-HBW experiment
  - Complicates evaluation of design metrics
- Strawman blended design
  - Design for 9-LBW experiment ensure all factors are included in design
  - Keep statistical advantages and design metrics of DOE
  - Assume “extra” data between target points enhances modeling
  - Assume blended design is obtained by removing redundant  $\alpha$ - $\beta$  targets
  - Blended designs require rig move slow enough to allow full sweep of controls at each  $\alpha$ - $\beta$  target point



# Blended DOE-L2F Design Metrics (9-factors)

Block Type	Blocks	Runs	Terms	VIF	% Power
	included	target points			$2\sigma$ , $s/n=2$
Factorial	1	134	Linear + 2FI	1*	99.7
FCD	1, 2	156	Quadratic	9.68	84.2
Nested FCD	1, 2, 3	312	Quadratic	22.47	86.7
I-optimal	1, 2, 3, 4	384	Quadratic	4.85	99.9

\*Squared factors are aliased

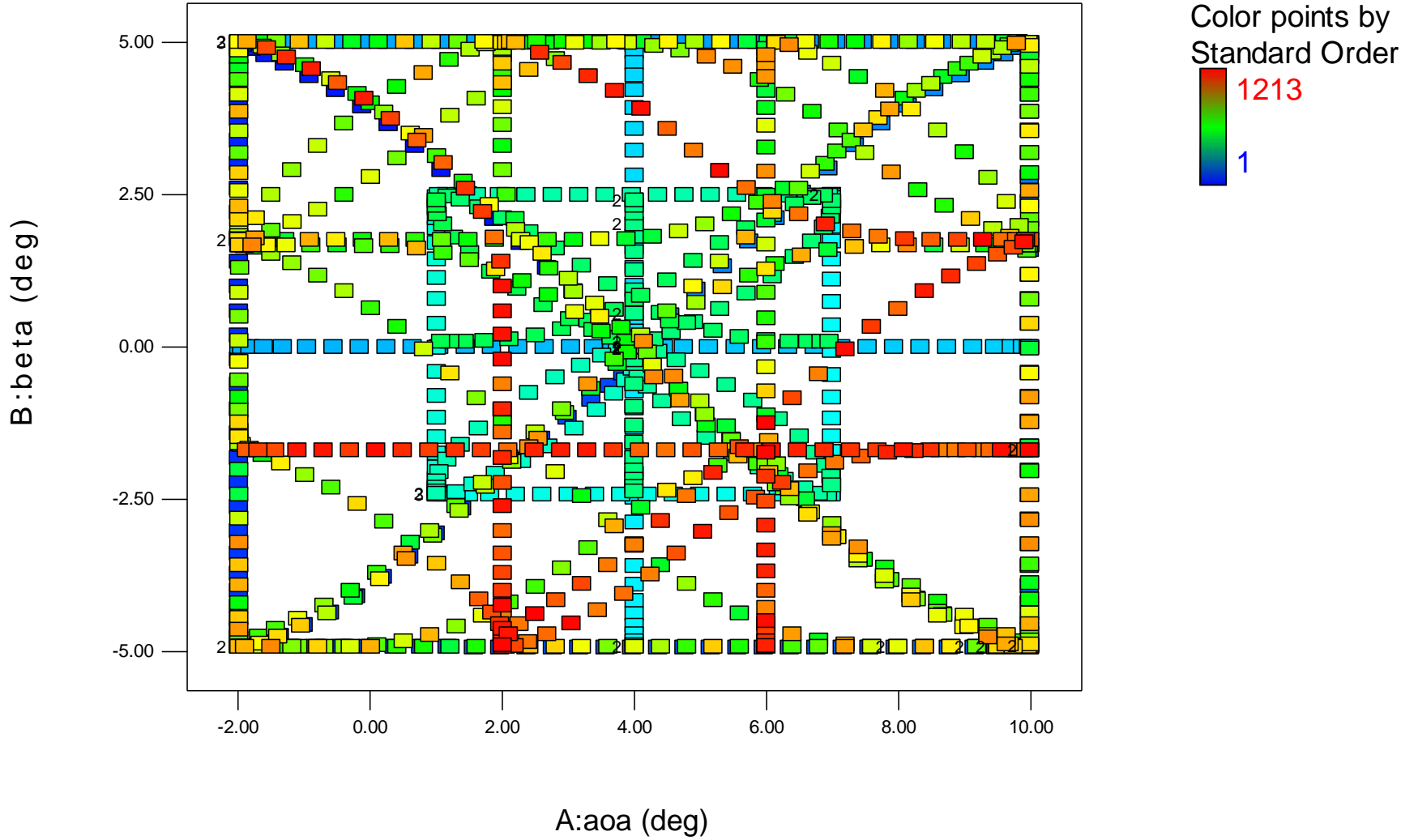
*Some Lessons Learned:*

*Fewer blocks required with continuous sampling*

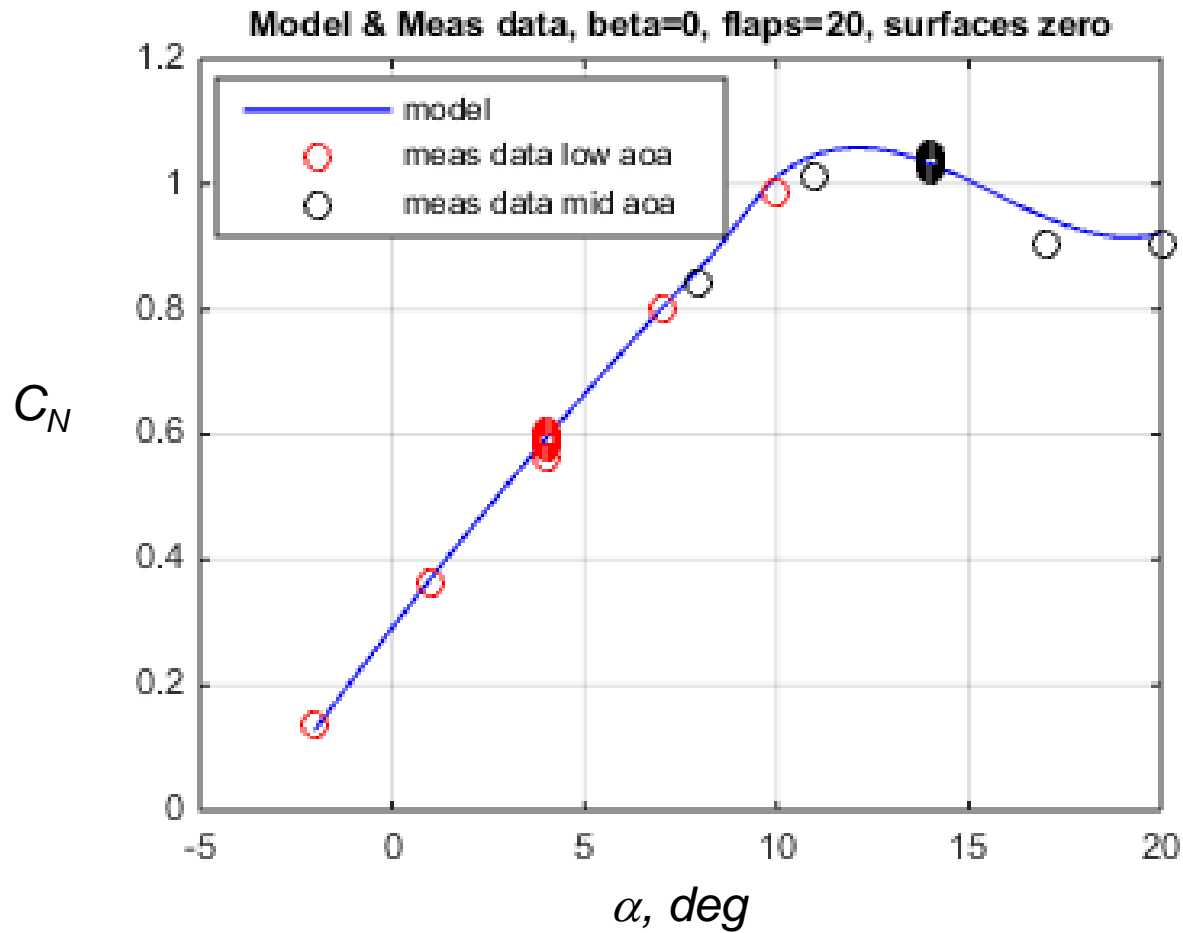
*Divide optimal blocks!*

*4<sup>th</sup> block provided too much data for the blended design.*

# Blended DOE-L2F Trajectories

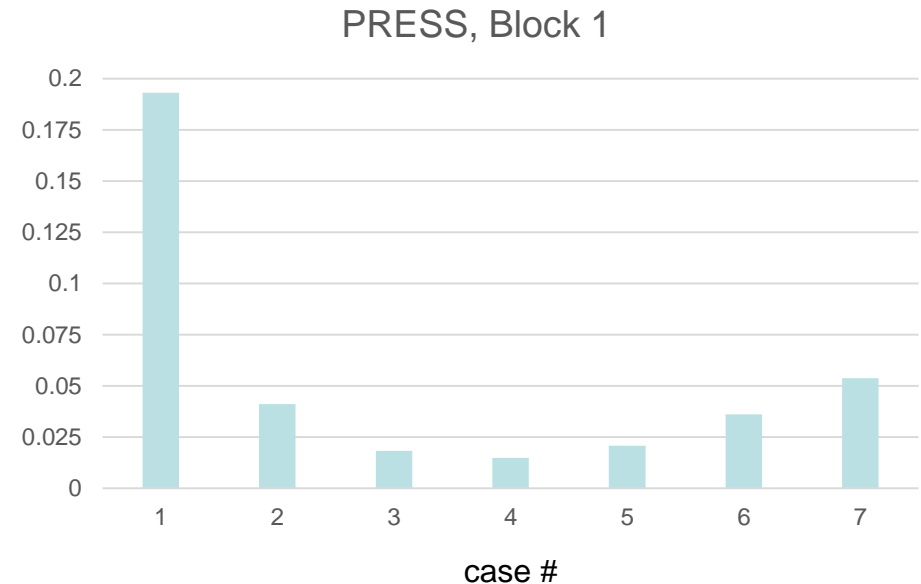


# DOE Model (3 blocks)



# DOE Modeling Progression

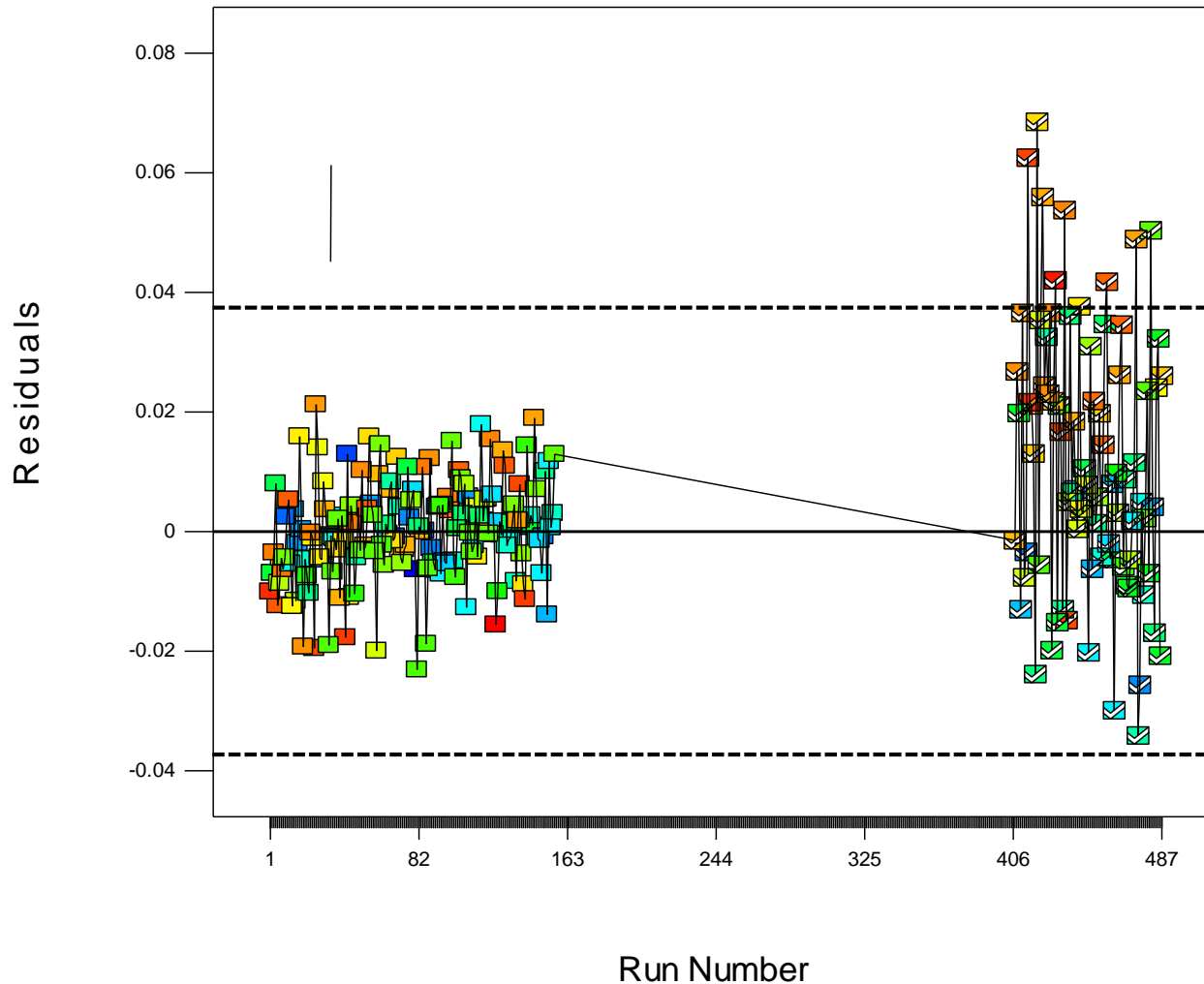
- Block 1 (FCD)
- 1<sup>st</sup> in series of sequential tests
- Case #2 – error budget satisfied
- Case #3 – best model is cubic
- Case #4 – minimum PRESS
- Case #6 – minimum Std. Dev



case #	1	2	3	4	5	6	7
block #	1	1	1	1	1	1	1
Design model	FCD	FCD	FCD	Nested FCD	I-Optimal	I-Optimal	I-Optimal
Model terms (No.)	Linear + 2FI (12)	quadratic + 2FI (20)	cubic + 3FI (32)	cubic + 3FI (38)	cubic + 3FI (68)	cubic + 3FI (81)	cubic + 3FI (128)
R <sup>2</sup>	0.9931	0.9988	0.9996	0.9997	0.9995	0.9996	0.9999
Std. Dev.	0.0351	0.0149	0.0095	0.0084	0.0064	0.0060	0.0060
PRESS	0.1931	0.0411	0.0183	0.0149	0.0208	0.0362	0.0538
**e <sub>i</sub> /C <sub>M</sub> max %	5.68%	0.22%	0.15%	0.11%	0.12%	0.10%	0.10%

\*residual  $e_i = C_{N\_measured} - C_{N\_predicted}$ , \*\* $C_{Mmax} = 1.22$

# Validation Test, DOE Block 1



- Residuals vs Run
- Block 1,  $\frac{1}{4}$  FCD
- $C_N$  low  $\alpha$  range
- Case #3 model
- 8 fail 3% error

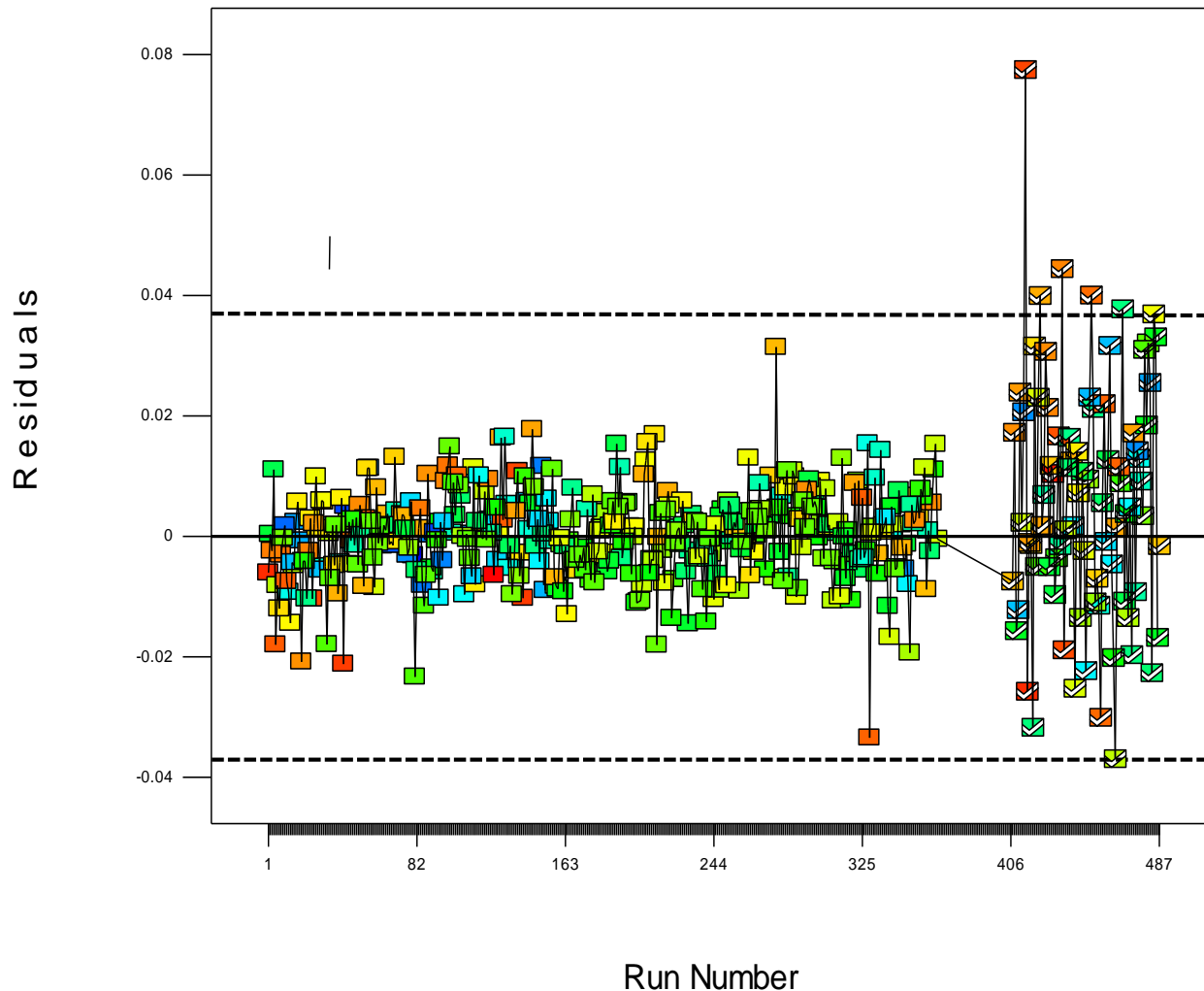
$\pm 3\%$  error

$\pm 3\%$  error test

- 8 points failed
- 9 of 81 allowed

*Validation tests reveal true prediction & bias errors*

# Validation Test, DOE Blocks 1-3

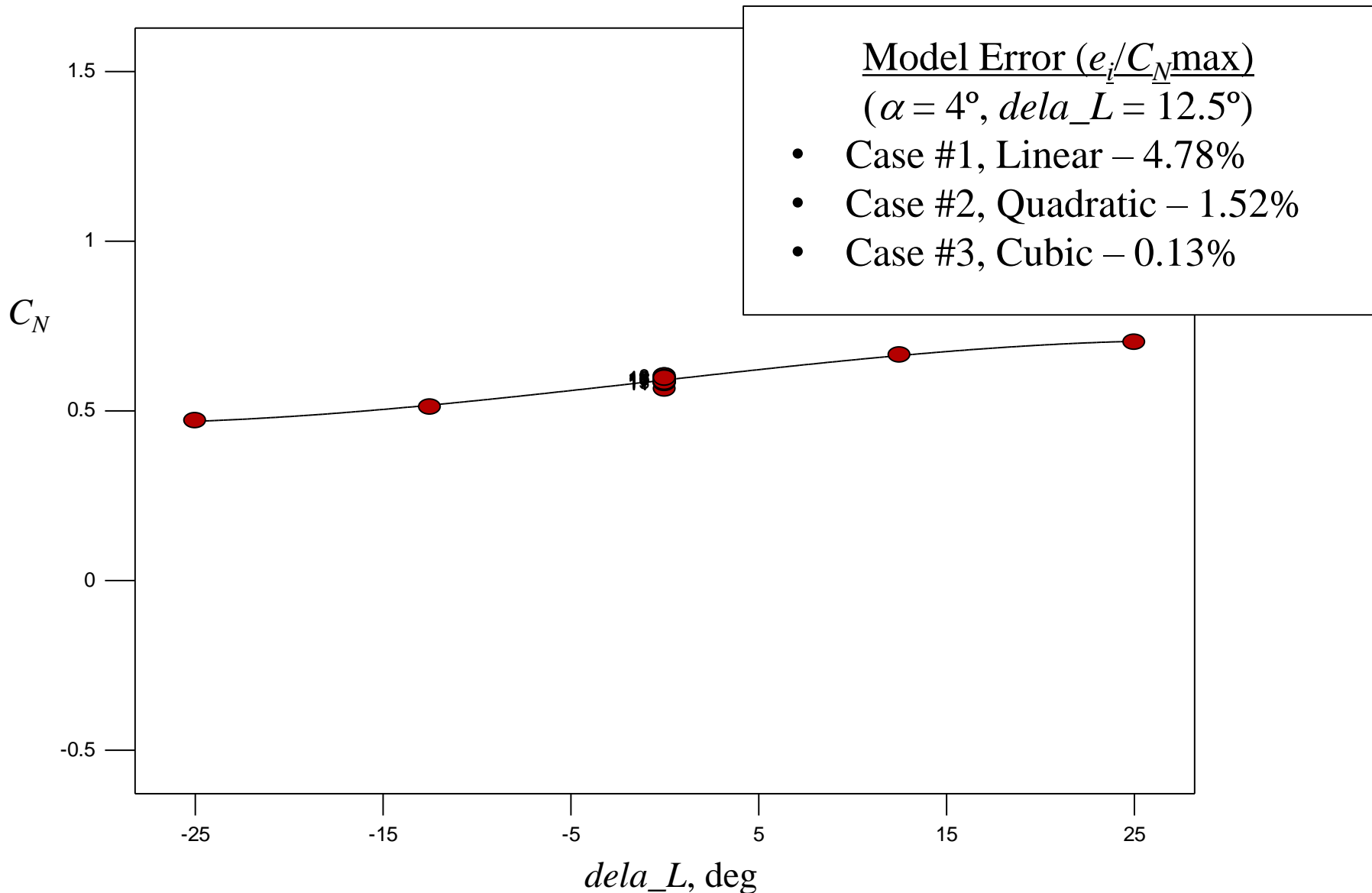


- Residuals vs Run
- Blocks 1-3, opt.
- $C_N$  low  $\alpha$  range
- Case #3 model
- Similar final stats

± 3 % error

***Model confirmed by validation test; 6 points fail 3% error test***

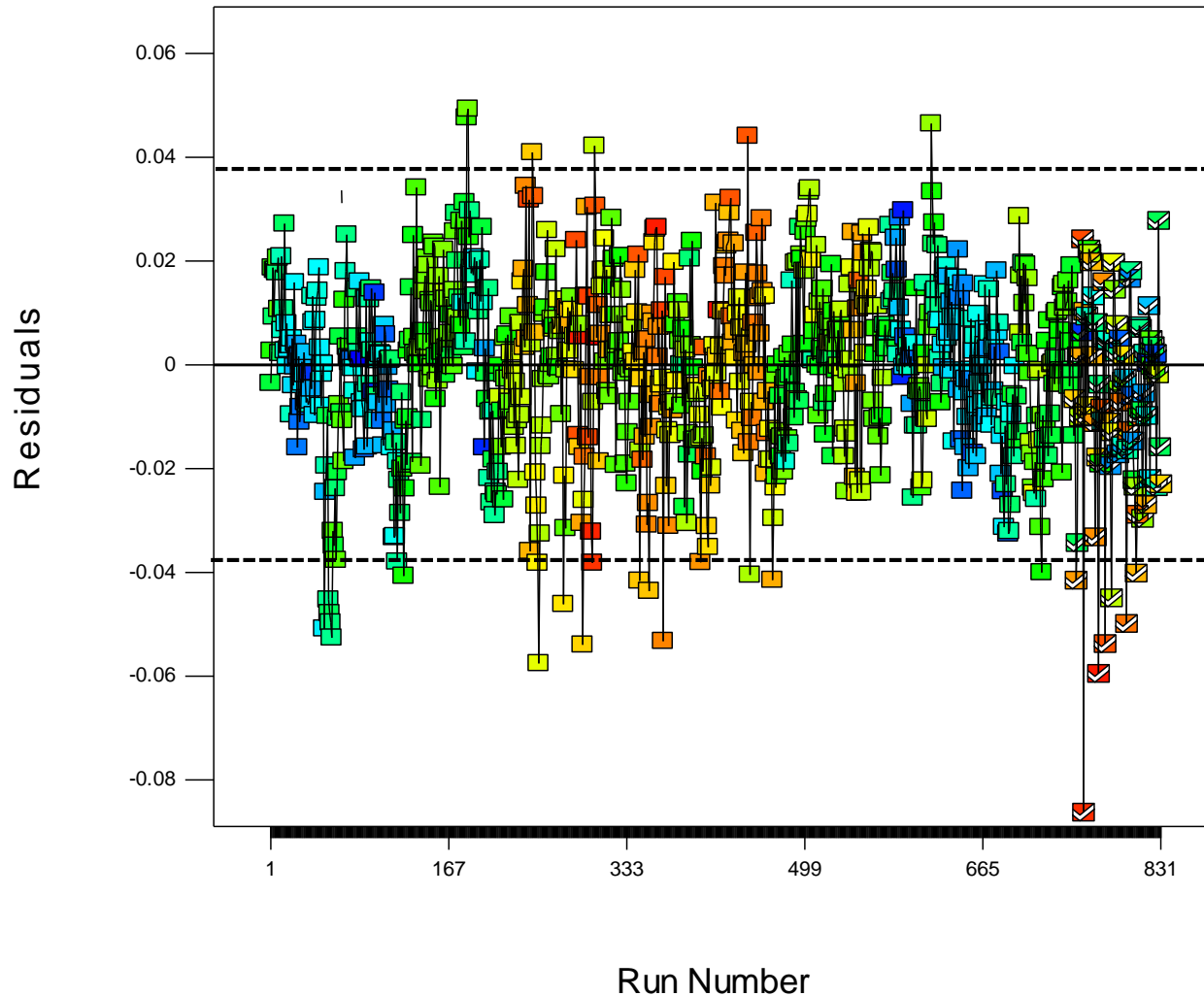
# Source of Cubic Terms (DOE blocks 1-3)



# ***L2F Test in LaRC 12-Foot Tunnel***



# Validation Test, L2F

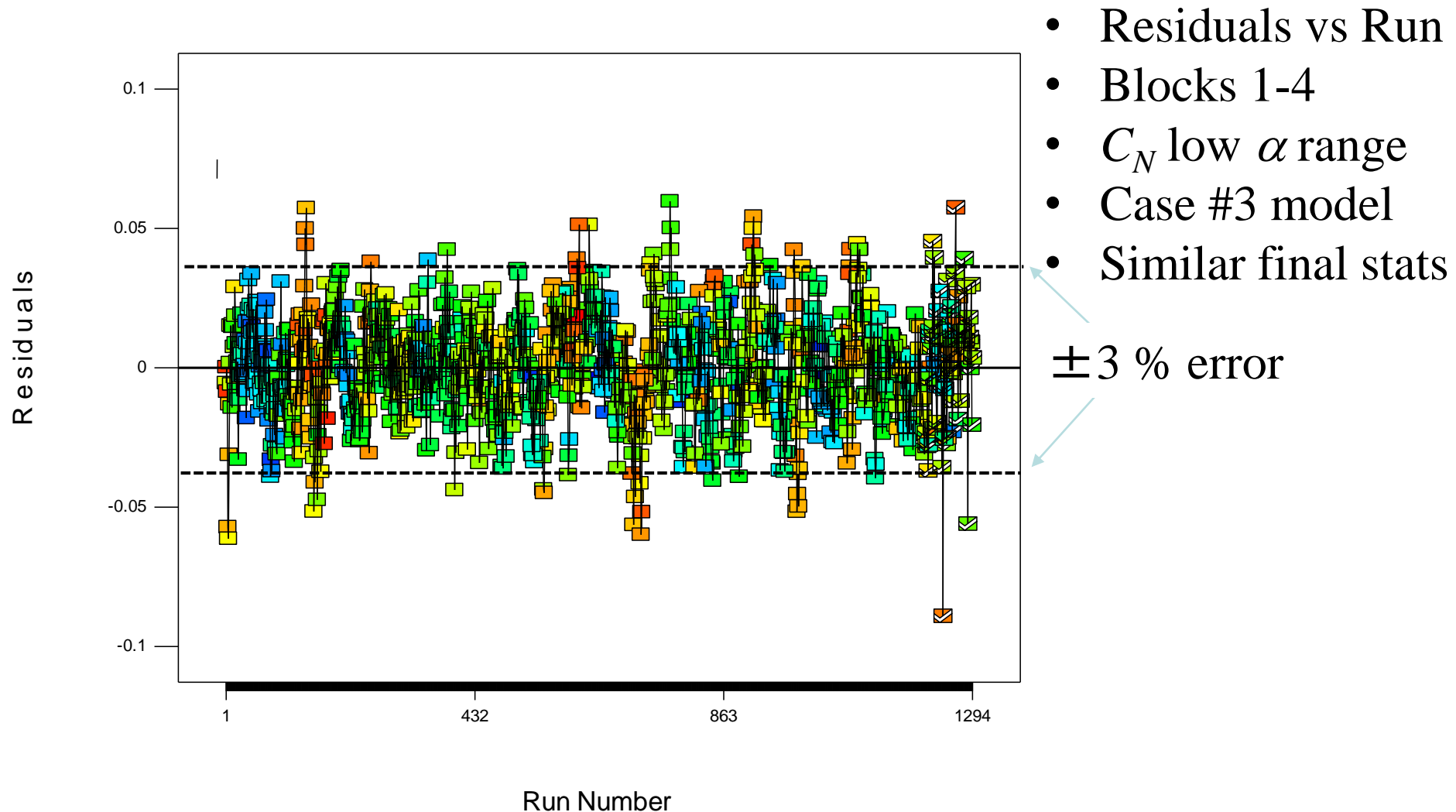


- Residuals vs Run
- Block L2F
- $C_N$  low  $\alpha$  range
- Case #3 model
- Similar final stats

$\pm 3\%$  error

***Model confirmed by validation test; 7 points fail 3% error test***

# Validation Test, Blended DOE-L2F



***Model confirmed by validation test; 6 points fail 3% error test***



# Concluding Remarks

- Sequential testing & validation recommended
  - Obtain data sequentially as required
  - Apply validation test after each block of data
- Efficient test methods demonstrated
  - DOE & L2F approaches provide methods to increase efficiency
  - Blending DOE-L2F
    - Currently a “work in progress” but shows promise
    - Presents a challenge in design phase to combine LBW+HBW factors
- Future Test Refinements
  - Fewer blocks required with continuous sampling
  - Smaller optimal blocks
  - Lower sample rates for “quasi-static” tests
  - For “quasi-static” case, lower bandwidth of HBW inputs
  - Design must reflect significant data added by HBW factors

# Questions?

- Contact Information

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- 757-864-1142

*“All models are wrong,  
but some are useful” –  
George E. P. Box*

